K24U 0061

Max. Marks: 48

Reg. No.:....

Name:.....

VI Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/
Improvement) Examination, April 2024
(2019 to 2021 Admissions)
CORE COURSE IN MATHEMATICS
6B13 MAT: Linear Algebra

Time: 3 Hours

PART - A

Answer any 4 questions. Each question carries one mark.

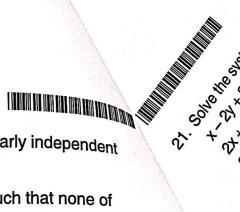
- 1. Define subspace of a vector space.
- 2. What is the dimension of the vector space of all 2×3 matrices over R?
- 3. State Dimension Theorem.
- 4. The characteristic roots of a matrix A are 2, 3 and 4. Then find the characteristic roots of the matrix 3A.

5. Find the eigen values of the matrix
$$A = \begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$$
.

Answer any 8 questions. Each question carries two marks.

- 6. Let $V = \{(a_1, a_2) : a_1, a_2 \in R\}$. Define $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, 0)$ and $c(a_1, a_2) = (ca_1, 0)$. Is V a vector space over R with these operations? Justify your answer.
- 7. Prove that the set of all symmetric matrices of order n is a subspace of the vector space of all square matrices of order n.





- 8. Check whether the set $\{(1, -1, 2), (2, 0, 1), (-1, 2, -1)\}$ is linearly independent or not.
- 9. Give an example of three linearly dependent vectors in R³ such that none of the three is a multiple of another.
- 10. Find the rank of matrix A, where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$.
- 11. Show that rank of a matrix, every element of which is unity, is 1.
- 12. Show that $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(a_1, a_2) = (a_1 + a_2, a_1)$ is a linear transformation.
- 13. Explain the condition for consistency and nature of solution of a non homogeneous linear system of equations AX = B.
- 14. Let $T: V \rightarrow V$ be a linear transformation. Find the range and null space of zero transformation and identity transformation.
- 15. Prove that the Eigen values of an idempotent matrix are either zero or unity.
- 16. Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.

Answer any 4 questions. Each question carries four marks.

- 17. Prove that any intersection of subspaces of a vector space V is a subspace of V.
- 18. Suppose that T: $\mathbb{R}^2 \to \mathbb{R}^2$ is linear, T(1,0) = (1,4) and T(1,1) = (2,5). What is T(2,3)? Is T one-to-one?
- 19. Let T : $R^2 \to R^3$ be defined by $T(a_1, a_2) = (a_1 a_2, a_1, 2a_1 + a_2)$. Let β be the standard ordered basis for R^2 and $\gamma = \{ (1, 1, 0), (0, 1, 1), (2, 2, 3) \}$. Compute $[T]^{\gamma}_{\beta}$.
- 20. Under what condition the rank of the following matrix A is 3? Is it possible for the rank to be 1 ? Why ? $A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$.



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21. Solve the system of equations.

$$x - 2y + 3z = 0$$

$$2x + y + 3z = 0$$

$$3x + 2y + z = 0$$

- 22. Find the eigen vectors of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.
- 23. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ find A^2 using Cayely Hamilton theorem and then find A^3 .

Answer any 2 questions. Each question carries six marks.

- 24. Prove that the set of all $m \times n$ matrices with entries from a field F is a vector space over F with the operations of matrix addition and scalar multiplication.
- 25. Find the inverse of $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ using elementary row operations.
- 26. Find the values of a and b for which the system of equations

$$x + y + z = 3$$

$$x + 2y + 2z = 6$$

$$x + 9y + az = b have$$

- 1) no solution;
- 2) unique solution and;
- 3) an infinite number of solutions.
- 27. Using Cayley Hamilton theorem find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.